

Poisson Distribution

So far so good, however, if you know how often something fails, how do you measure how likely it is to fail tomorrow? This is where poisson and some fairly serious maths comes in:-

Thankfully the maths isn't required in the syllabus, just an explanation of poisson distribution, what a relief ... unless you're into stats in a big way.

First of all the rules :-

You need a large sample, and you need to be looking for something with a low probability, such as accidents at work in a large workforce and the probability that one person will have a lot of them (i.e. they are accident prone)

Some new symbols for this :-

$e=2.7183$ base of natural (napierian) logarithms

m = mean

$r!$ = factorial r so $3!$ is $3 \times 2 \times 1 = 6$ (factorial 0 is 1)

The probability of r accidents is represented by the poisson formula:-

$$p(r) = \frac{e^{-m} m^r}{r!}$$

So an example

on average there are 8 lost time accidents per year in a workforce of 70 people, to the average accident rate m is $8/70 = 0.115$ and we want to know the probability that one person will suffer three accidents in a year.

$$p(3) = \frac{2.7183^{-0.115} 0.115^3}{3!} \quad \text{Which evaluates to} \quad p(3) = \frac{0.8194 \times 0.0015}{6} = 0.0002$$

To make a poisson distribution graph (and therefor find the highest probability result) you would repeat that calculation for a range of r values and plot the results in a histogram or line graph.

Number of accidents	Probability of that number
0	0.8914
1	0.1025
2	0.0059
3	0.0002

Lets take that over 48 years (average working life being 17-64)

The average we have is per year, so for 48 years we should multiply this by 48.

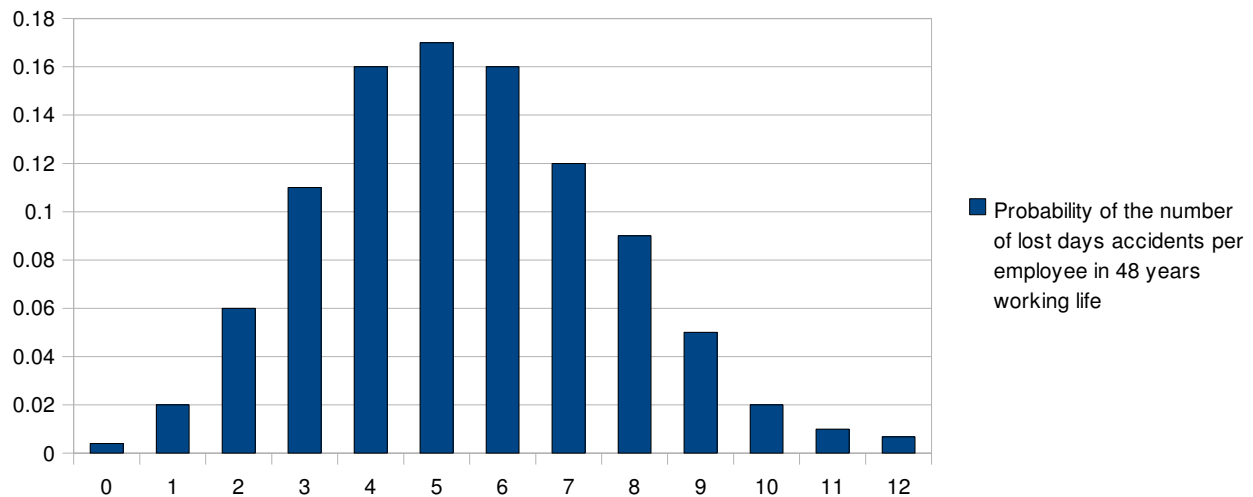
$0.115 \times 48 = 5.52$, now lets plug that into a poisson calculation.

The formula now looks like this
$$p(r) = \frac{2.7183^{-5.52} 5.52^r}{r!}$$

We'll run the calculation for a range of Zero to 12 accidents and find the probabilities of each result.

Number of accidents	Probability of that number
0	0.0040
1	0.0221
2	0.0610
3	0.1123
4	0.1550
5	0.1711
6	0.1574
7	0.1241
8	0.0856
9	0.0525
10	0.0290
11	0.0150
12	0.0067

So lets see that as a graph



You can see the graph is not symmetrical, and that the modal value (the most common value) is 5, but the average is we know 5.52

From this we can say that an employee is likely to have between 3 and 8 lost time accidents, the probability of an employee getting away unscathed or having 12 lost time accidents is very low.